Azonosító jel: | | | | | | | | | | | | | | | |

ÉRETTSÉGI VIZSGA ● 2008. május 6.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

2008. május 6. 8:00

Az írásbeli vizsga időtartama: 240 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

OKTATÁSI ÉS KULTURÁLIS MINISZTÉRIUM

Important information

1. The exam is 240 minutes long, after that you should stop working.
2. You may solve the problems in any order.
3. In Section **II**, you are only required to solve four out of the five problems**. Please remember to enter the number of the question you have not attempted into the empty square below.** Should there *arise any ambiguity* for the examiner whether the question is to be marked or not, it is question no. 9 that will not going to be assessed.
4. You may work with any calculator as long as it is not capable of storing and displaying textual information and you may also consult any type of four digit mathematical table. The use of any other kind of electronic device or written source is forbidden.
5. **Remember to show your reasoning, because a major part of the score is given for this component of your work.**
6. **Remember to outline the substantial calculations**.
7. When you refer to a theorem that has been covered at school and has a common name (e.g. Pythagoras' theorem, sine rule, etc.) you are not expected to state it meticulously; it is usually sufficient to put the name of the theorem, however, you are supposed to show why does it apply. Any reference to any other theorem, however, can be accepted only if it is stated exactly with all the conditions (proof is not required) and you explain how it applies in the given situation.
8. Remember to answer each question (i.e. providing the result) also in textual form.
9. You are supposed to work in pen; diagrams, however, may also be drawn in pencil. Anything written in pencil outside the diagrams cannot be evaluated by the examiner. Any solution or some part of a solution that is crossed out will not be marked.
10. There is only one solution will be marked for every question. If you attempt a question more than once then you should **clearly indicate** the one to be marked.
11. Please, do not write anything in the shaded rectangular areas.

I.

1. Let a_1, a_2, …, a_{21} be the first twenty one terms of an arithmetic progression. The sum of the terms at the odd positions is greater by 15 than the sum of the terms at the even positions. It is also given that $a_{20} = 3a_9$. Find the value of a_{15}.

T.:	12 points	

2. There were 100 students from the 9-12th grades of a Hungarian high school subject to an international survey on math education. Each student took the same test and the maximal score was 150 points. The average score of the students was 100 points. The number of students from the grades 9-10th was one and a half times of the number of students from the grades 11-12 th and, at the same time, the mean score of the students from the grades 11-12 th was one and a half times of the mean score of the students from the grades 9-10 th.

***a*)** Calculate the mean score of the students from the grades 11-12 th.

Having run the survey the organizers were also interested about how did the students judge the diffuculty of the questions. There were three students out of the 100 chosen randomly and they were asked to fill a questionaire.

***b*)** What is the probability that there were 2 students chosen from the grades 9−10 th and 1 student from the grades 11−12 th?

a)	7 points	
b)	5 points	
T.:	12 points	

3. Determine the value of the real parameter α in such a way that the equation

$$4\cdot x^2 - 4(\sin\alpha + \cos\alpha)\cdot x + 1 + \sin\alpha = 0$$

has a double real root.

T.:	13 points	

4. There are 10500 students studying at three faculties of the university, altogether. Three candidates, Alchemist, Owl and Flute entered for the Student–Dean's illustrious position. There were 76% of the students who actually voted and having processed 90% of the votes the following results were announced by the local radio station College Voice: so far there were 2014 votes for Alchemist, 2229 votes for Owl and 2805 votes for Flute.

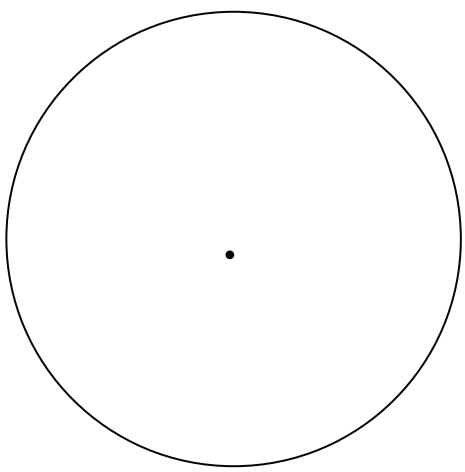
a) What percent of the ballots processed so far was spoiled? (The answer should be given correct to one decimal place.)

b) Draw, on a pie-chart, the percentage distribution of the processed votes, also indicate in degrees the rounded measures of the central angles corresponding to each region, respectively. (The percentages and the angles should be rounded to the nearest integer.)

c) Is it still possible for Alchemist to win the election? (The winner is the candidate who gets the highest number of votes.)

d) By at least how many percents should Flute be ahead of the next candidate after having counted the 95% of the votes to feel secure, by mathematical certainty, about his victory? (The minimal value of this percentage should be given, rounded to one decimal place.)

a)	3 points	
b)	4 points	
c)	3 points	
d)	4 points	
T.:	14 points	



II.

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

5. Andrew and Bernie are training in an alpine camp. They are running 10 kilometers every single morning: 5 kms up to the peak and then, without a break, 5 kms back to the camp, on they same way down. One morning Andrew parted 10 minutes before Bernie. His average speed was 15 km/h upwards, and 20 km/h downwards. Bernie's speed was 16 km/h upwards and 22 km/h downwards this morning.

a) At what distance, from the peak of the mountain, did they meet on their course this morning?

There were 10 girls and 9 guys in the camp, altogether. On the very first session the coach was asking everyone about the number of their former acquaintances in the group. (Acquaintance is a symmetric relation.) We know that each guy knew the same number of girls from before, but there were no two girls who reported the same number of boys among their former acquaintances.

b) Is it possible that each boy knew 6 girls from before at the beginning of the camp?

a)	10 points	
b)	6 points	
T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

6. Given is a mirror symmetric and tangential trapezium i. e. that has an inscribed circle. The lengths of its bases are 5 and 20 units, respectively.

a) Calculate the area of the trapezium and also the length of its diagonal.

b) Calculate the volume of the solid of revolution obtained by rotating the trapezium about its longer base.

c) Prove the following statement:
If a trapezium is cyclic and tangential at the same time then its height is equal to the geometric mean of the two bases.

a)	5 points	
b)	5 points	
c)	6 points	
T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

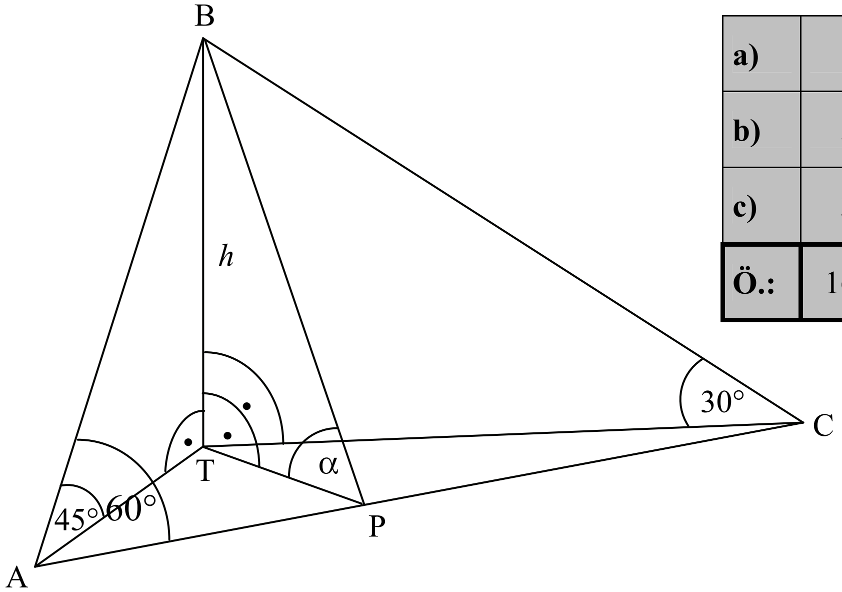
7. A meteorological balloon was launched at the seaside a few minutes before noon. While emerging the balloon was drifting towards the sea. It was 12 o'clock sharp and the altimeter of the balloon was indicating 842 meters when Andrew and Cecil, both somewhere on the seaside managed to measure the position of the balloon with their protractors, respectively. Andrew found that the angle of elevation of the balloon's position (the angle made by the horizontal plane) was 45°, while the rays from the balloon and Cecil's position on the seaside were subtending an angle of 60° at Andrew's point of observation. At Cecil's position, however, the angle of elevation of the balloon was 30°.

a) Find the distance of the two instruments.

b) There is a certain point *P* somewhere on the segment between the positions of Andrew and Cecil, from which, at 12 o'clock, the angle of elevation of the balloon was maximal. Prove that *P* is the foot of the height from *T* of the triangle *ACT*.

c) What was the balloon's altitude at 12: 30 when its barometer was displaying 80% of the sea level pressure?

The athmospheric pressure, in terms of the altitude above sea level can be calculated according to $p(h) = p_0 e^{Ch}$. Here *h* stands for the altitude above sea level in meters, p_0 is the pressure at sea level (this one can be assumed to be equal to 10^5 Pascal), *e* is the base of natural logarithm ($e \approx 2{,}718$), finally, *C* is an empirical constant ($C = -\frac{1}{7992}$).



a)	8 points	
b)	5 points	
c)	3 points	
Ö.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

8. The editor of a publishing house is designing the layout of a book. The planned margins are 2 cm wide on the top, on the bottom and on the outer edge of the pages, but the inner margins have to be 4 cm wide because of the binding. The total area of a page is 600 cm^2.

a) How should the editor set the dimensions of the pages in order to have the maximal printing area?

b) There are 120 printed pages in the book and the page numbering starts from 3. Choosing a printed page randomly what is the probability that its page number contains the digit 2?

a)	12 points	
b)	4 points	
T.:	16 points	

You are supposed to answer any four of the questions no. 5-9. The number of the question not attempted should be entered into the empty square on sheet no. 3.

9. There were six PhD students and three professors, a biologist, a physicist and a mathematician, to be awarded the Medal of Excellence for their outstanding scientific achievement at the closing ceremony of the University's Faculty of the Natural Sciences. There were 9 seats booked for them in the frist row, next to each other. The professors arrived to the ceremony well ahead of the six students.

a) How many ways would there be for the professors to occupy three of the nine empty seats if they were not to wait for the arrival of the students?

The professors, however, were waiting patiently until all of the six students have arrived. Then the professors requested the students if it were possible for each of them to sit down between two students. The students, of course, were well pleased to accept the proposal.

b) How many ways are there to arrange the 9 guests of honour according to this request?

c) What is the probability that the professor of Biology is called second to receive the Medal, and, additionally, both before and after him there is a PhD student called to receive their Medals, respectively. (Assume that each order of calling the guests is equally probable.)

a)	4 points	
b)	6 points	
c)	6 points	
T.:	16 points	

	number of question	maximal score	score	maximal score	score
PART I	1.	12		**51**	
	2.	12			
	3.	13			
	4.	14			
PART II.		16		**64**	
		16			
		16			
		16			
		← problem not chosen			
			TOTAL	**115**	

date

examiner

	score attained (elért ontszám)	score input for program (programba beírt pontszám)
Paper I/(I. rész)		
Paper II/(II. rész)		

date /(dátum)

date /(dátum)

teacher/(javító tanár)

registrar/(jegyző)